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Single file diffusion enhancement in a fluctuating modulated quasi-1D channel

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Abstract – We show that the diffusion of a single file of particles moving in a fluctuating modulated quasi-1D channel is enhanced with respect to the one in a bald pipe. This effect, induced by the fluctuations of the modulation, is favoured by the incommensurability between the channel potential modulation and the moving file periodicity. This phenomenon could be of importance in order to optimize the critical current in superconductors, in particular in the case where mobile vortices move in 1D channels designed by patterns of pinning sites.

Introduction. – Superconducting devices have attracted growing interest with the recent advances of lithography techniques in which pinning sites can be created and controlled, so as to pin the vortices and thus increase the critical currents. Therefore, many studies have been focused in the last past years on the properties of a mobile file of vortices moving in quasi-1D channels. Some are devoted to the determination of the mobility of vortices moving in a strip geometry in order to investigate the influence of this motion on the generation of noise [1,2], others explore the driven vortex flow in modulated channels designed by pinned vortices [3–7]. This latter case is a particular aspect of the general issue of the driven transport of a 2D flux lattice submitted to a random [8–12] or periodic [13–18] pinning potential, which has been given increasing attention for some years. In quasi-1D channels, the vortex mobility depends upon the adequacy between the inter-vortex distance and the periodicity of the pinned vortices array and/or the channel width. For instance, it is well known that this mobility drops when the vortices file diffuses in a channel with a commensurate configuration of the pinning sites.

Experimentally, these matching effects have been observed on the macroscopic scale at which they result in an important increase of the critical current for some precise values of the magnetic field [3]. These observations have been confirmed by simulations [4–7]. In these situations, the modifications in the critical current have been associated with the variation of the net velocity of the particles, viz. the first moment of the velocity distribution, which is non-zero because of the presence of a driving force. Note that, even in the absence of this driving force, the existence of a superconducting state or, more generally, the quality of vortex transport, still depend on the fluctuations around this mean velocity, viz. the second moment of the velocity distribution, as discussed in ref. [19]. In this letter, we focus on the evolution with time of such quantity, that is to say on diffusion processes.

As for a single particle, the diffusion of a strictly 1D file of particles on a periodic substrate is lowered relatively to the free case [20]. However, an interesting phenomenon is the non-monotonous evolution of the mobility factor when an increasing constant force is applied. As for a single particle [21,22], the diffusion can be greatly enhanced when an adapted non-zero force is applied, but drops when this force is too important. This shows that the dependencies of the mean-square displacement and velocity on the environment of the file of particles can be quite complex. Such rich behaviour can also be met when ac forces are applied [23,24]. Similarly, in quasi-1D channels that can be found in pinned lattices, the diffusion of the mobile file could be influenced by the surrounding pinned vortices, as well as the net current is. More precisely, the high sensitivity of the critical current upon the ratios between the characteristic lengths of the problem, that is exhibited in refs. [4–7], suggests that

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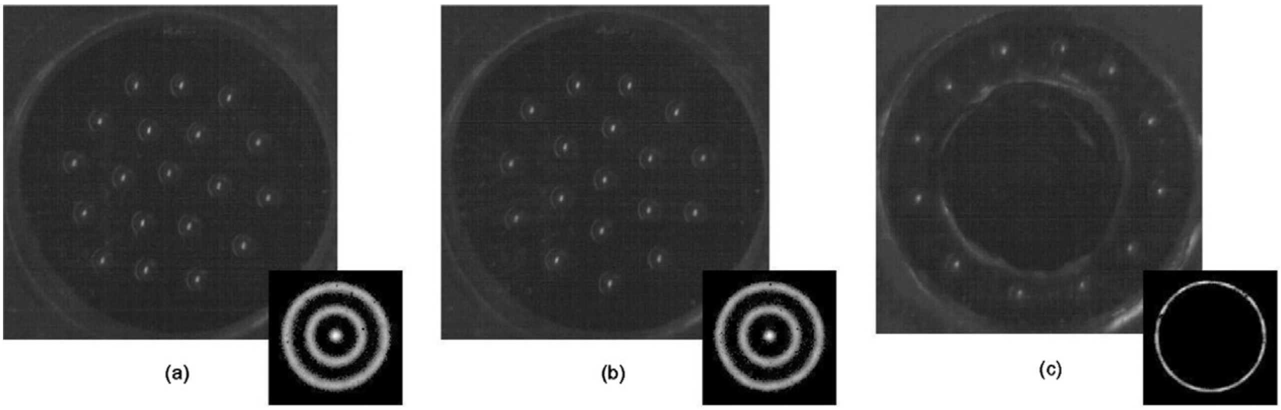

Fig. 1: Experimental configurations. Wigner islands: (a) $N=19$, (b) $N=18$; circular bald pipe (c) $N=12$. In inset, the corresponding balls trajectories at the same effective temperature.

fluctuations in the channel modulation, originating in the small movements of trapped vortices for instance, could subtly influence the resulting flow of the mobile vortices.

In this letter we show that these fluctuations could induce an important enhancement of the vortices diffusion, one order of magnitude higher than in the case of a bald channel. This enhancement is shown on a macroscopic experimental device, which allows to determine directly the trajectories of the particles. The diffusion of a circular file of particles has been studied for two different channels guiding the particles: a fluctuating modulated channel created by other particles, and a channel without any modulation, hereafter called bald pipe, that was already considered in ref. [25].

Experimental set-up. – In these experiments, millimetric stainless-steel balls are located on the bottom electrode (silicon wafer) of a horizontal plane condenser, while a metallic frame intercalated between the two electrodes and in contact with the bottom electrode confines them. Depending on the experiments, this frame is a disk whose external diameter is 10 mm or a circular bald pipe, its external diameter of 10 mm and its width of 2 mm preventing any crossing between particles (fig. 1). When a voltage $V=1\,\text{kV}$ is applied between the two electrodes, the balls become charged, repel each other and spread throughout the whole available space. We have shown that their electrostatic interaction is described by a modified Bessel function of the second kind K_0 with a screening length $\lambda=0.48\,\text{mm}$. Notice that this interaction is exactly similar to the inter-vortex interaction in superconductors [26,27]. To introduce thermal noise, the whole cell is fixed on loudspeakers supplied by a white noise voltage. We have thoroughly checked that the resulting shaking of the balls, due to friction with the bottom electrode, fulfil the properties one can expect for a thermal shaking. First, even though all the balls lie on the same solid substrate, their movement is spatially non-correlated, which might be due to inhomogeneities on the wafer at a microscopic level. Secondly, the individual trajectory of a single ball which is free or trapped in a parabolic well can be described through Langevin formalism [25]. Stationary states are reached in a few tenth of second. They are characterized by an effective temperature directly controlled by the shaking amplitude: the energy distribution is given by Boltzmann statistics. This was proved on confined small islands of balls, that can be seen as two-level systems when considering their two first equilibrium configurations, characterized by concentric shells of varying number of balls [28]. This effective temperature was calibrated and is measured *in situ*.

Throughout the experiments, images of the particles are recorded in real time using a camera. The interval between two successive snapshots is 150 ms and five series of 10000 images have been recorded for each experiment. With this choice, relevant statistics for the long-time behaviour of the displacements are obtained, the length of one experiment remaining reasonable since the effective thermal bath is characterized by a relaxation time of about 100 ms [25]. The diffusion of the particles is measured through the evolution with time of their mean-square displacements (m.s.d.) $\Delta\theta^2(t)$ given by

$$\Delta\theta^2(t)=\langle\left[\theta(t+t_0)-\theta(t_0)-\langle\theta(t+t_0)-\theta(t_0)\rangle\right]^2\rangle, \quad (1)$$

where the orthoradial coordinate θ (in radians) is the cumulated angle, and not the modulo 2π angle in order to explore an unbounded motion. The brackets $\langle\,\rangle$ denote ensemble averaging over the initial time t_0 and a set of statistically independent trajectories.

Observations and discussion. – In order to observe a quasi-1D movement in a fluctuating modulated potential, we first considered the outer circular shell of a Wigner island, composed of 19 interacting particles confined in a circular disk, for which the ground configuration corresponds to self-organized pattern constituted of three concentric shells filled by 1, 6 and 12 balls. This configuration is denoted (1-6-12) hereafter (fig. 1) [27]. To observe

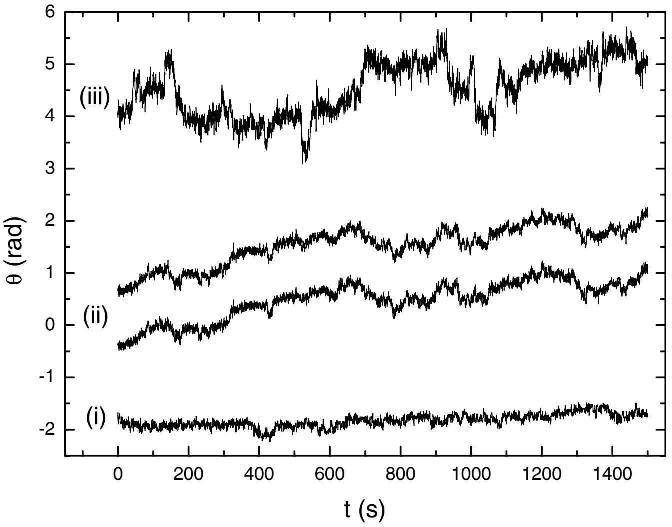

Fig. 2: Evolution with time of the orthoradial coordinate θ of a ball in: a bald circular pipe (i), a Wigner island $N=19$ outer shell (two different balls) (ii), a Wigner island $N=19$ inner shell (iii).

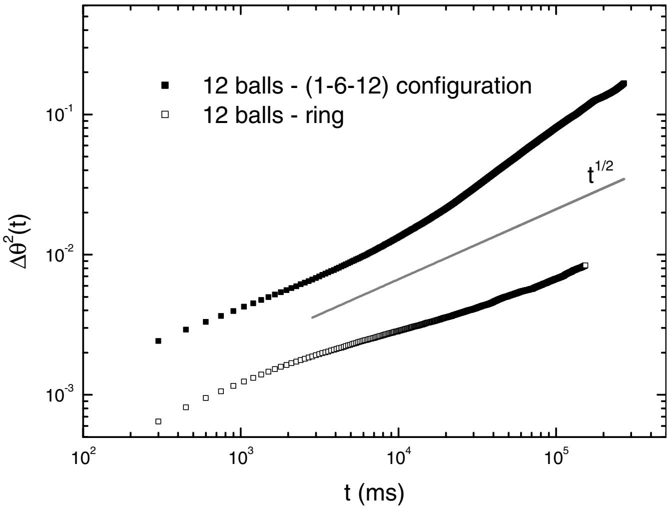

Fig. 3: Comparison between the mean-square orthoradial displacements $\Delta\theta^2(t)$ of a ball moving in the Wigner island $N=19$ outer shell and in a bald circular pipe (log-log scale).

well-defined shells, the experiments have been performed at an effective temperature equal to roughly a few tens of the inter-ball interaction; for this temperature range, the balls seldom jump from one shell to another [28]. The effective thermal agitation mainly induces orthoradial motion while the radial displacements are reduced as can be seen on the balls trajectories presented in the insets of fig. 1 [28].

This quasi-1D orthoradial displacement is the first step towards complete "melting" of such clusters, which is achieved when thermal fluctuations also break the radial order. For this reason, the influence of the temperature and the geometry on the orthoradial movement has been previously extensively studied (see, *e.g.*, refs. [28–31] and references therein). However, since the relevant parameter in that case is the orthoradial order, which can be appreciated by short-time measurements, long-time behaviour in the diffusion process has not been investigated in such systems. In particular, even when evolutions with time are reported, single file behaviours are not studied [32,33]. Let us also indicate that in our system, the orthoradial displacements grow monotonously with temperature [28], unlike what is observed in colloidal systems, where an orthoradial stabilization can be obtained when the radial fluctuations increase [32,33]. As shown by Schweigert *et al.*, this phenomenon is due to the presence of a hard-wall confining potential [34], which is not the case here [26].

The shell stability offers the opportunity to perform long-time orthoradial diffusion experiments. In the outer shell, the relative mean orthoradial displacements of two neighbouring balls are about 0.07 radian (respectively 0.15 in the inner shell), much smaller than the angular distance $2\pi/12$ (respectively $2\pi/6$) between them. Thus, each shell can be considered as a periodic ring. This ring presents global angular movements which are coherently followed by the balls as shown in fig. 2 (ii), where the trajectories of two different balls of the outer shell are reported. Their global orthoradial movements are similar but their trajectories present differences of low relative amplitude at small time-scale. Therefore, the outer twelve-ball shell is a well-adapted realization of a periodic system of particles moving in a fluctuating modulated potential due to the inner shell. Along the orthoradial direction, the angular displacements of the balls have a Gaussian distribution. The evolution with time of the corresponding m.s.d. $\Delta\theta^2(t)$ is presented in fig. 3 for $150\,\text{ms} \leqslant t \leqslant 150\,\text{s}$. Within this time range, this angular m.s.d. increases with time with a t^{α}-dependency where α is close to 0.6. Note that this power law description is only a useful tool to qualify the general trend of the curve, without possible extrapolation on the behaviour at longer times.

We have compared these results with those obtained for the same twelve balls moving in a circular bald pipe, the effective temperature and the inter-ball interaction remaining the same (fig. 1). We emphasize that, even if the movement of the balls is not strictly 1D, no coupling effects resulting from the circular channel geometry were observed. For instance, the orthoradial movement of a single ball in this circular gutter is a 1D free diffusion well described by the Langevin equation [25]. As in the Wigner island case, the distribution of the angular movement of the balls is Gaussian. However, long-time behaviour of the angular m.s.d. in this bald pipe differs dramatically from the previous case, since the amplitude of the diffusion is much smaller; 10 times smaller for instance for the last decade. This important difference in the mobilities can be also directly observed on the various trajectories presented in fig. 2 (i-ii). Moreover, the role of the periodicity is confirmed when considering the diffusion in the inner shell, which is around seven times higher than in the outer shell,

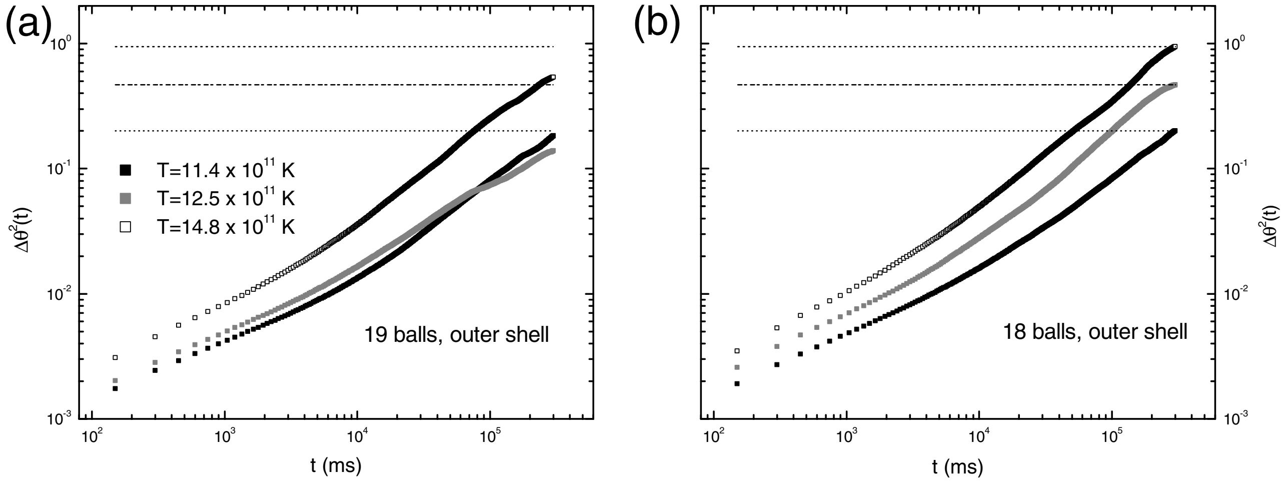

Fig. 4: Evolution with time of the mean-square orthoradial displacement of a ball located in the outer shell of (a) the commensurate Wigner island $N=19$ and (b) the incommensurate Wigner island $N=18$ (log-log scale). Broken horizontal lines are only useful guides that mark the displacements for $N=18$ and $t=t_{max}$ in order to compare the displacements in both situations.

with the same power law. Whereas an outer ball moves in a modulated potential with a $2\pi/6$ period, an inner ball is indeed submitted to a potential with a period twice as small; thus the impulsion transfer that enhances the diffusion is linked to the modulation of the potential.

One can try to understand this difference after noticing that in the bald pipe the movement is subdiffusive and characterized by a growth much slower than in the modulated case, α varying from 1/2 until $\alpha=0.4$ at the end of the experiment, as shown in fig. 3.

These power law behaviours must be discussed in the frame of the single file diffusion theory which describes the diffusion of particles in a single channel where the crossings are forbidden. Beyond the case of interacting vortices in superconductors, single file diffusion processes are encountered in various systems such as nanoporous materials [35–38] or colloidal systems [39–41]. For infinite systems, the theory predicts that correlations between particles induce a subdiffusion with $\alpha=0.5$ whereas classical free diffusion is characterized by $\alpha=1$ [42–45]. We suggested that the smaller value of α for the diffusion in the bald pipe resulted from the cyclicity of the system associated to the non-linearity of the interaction [25]. From this point of view, the $t^{0.6}$ behaviour observed in the outer shell of the Wigner island is all the more striking. It can be considered as an indication that the movement is not strictly 1D, viz. some slight radial shifts can allow the particles to be "closer" when considering the orthoradial projection of the positions. This could explain why we observe a diffusion that can be described, on the explored decades, by a t^α law with $\alpha \geqslant 0.5$, even though the diffusion is still abnormal because the crossing of the particles is not possible. Note that the exponents 0.4 or 0.6 are close to 0.5, which could suggest the observed discrepancies are irrelevant. However, as shown in fig. 4, an exponent $\alpha \geqslant 0.6$ is found whatever the conditions and is measured on two decades, which is more than in most experimental studies, then it can be trusted. Nonetheless, since we do not have any clear explanation for this difference with the common subdiffusive $t^{0.5}$ behaviour, we will just consider this trend as an indication of a non-totally 1D movement.

Similarly, the enhancement of the diffusion could be associated with this possibility of small lateral explorations which would become more important when a particle is situated in front of a minimum of potential. This also shows that the dynamics in pinned lattices cannot be interpreted through the sole conclusions of strictly 1D considerations. However, the possibility of small radial movements is not sufficient to explain the observed enhancement, since such movements are also allowed in all experimental systems where true subdiffusion with $\alpha \leqslant 0.5$ was observed [25,39–41]. In a recent paper, Bandyopadhyay *et al.* [46] have shown that diffusion of a single particle in a modulated potential can be largely enhanced if the particle is excited by a rapid fluctuating force. In the same way, we can attribute the diffusion increase to the fluctuations of the modulated potential felt by the outer balls, the fluctuating part of the force being associated with the momentum transfers resulting from the oscillations of the balls located in the inner shell.

To reveal how the channel modulation and its fluctuations can play a role, we have performed diffusion experiments in which we have modified the inter-shell interaction without drastically changing the moving file. We have compared the diffusion of the outer shell of the $N=19$ system to the one obtained with a $N=18$ island whose ground configuration is (1-6-11). The advantage of these two systems is that they roughly have the same number of balls in the outer shell whereas their relative symmetries strongly differ: the commensurate system $N = 19$ exhibits a threefold symmetry whereas the $N = 18$ system is incommensurate. This difference of relative symmetry between the two shells modify the inter-shell coupling and thus might influence the effect of this coupling on the diffusion. The variations of the corresponding m.s.d. $\Delta\theta^2(t)$ are presented in fig. 4 for three different temperatures T. For long times, the m.s.d. associated with the incommensurate $N = 18$ system increases roughly with the same power law ($\alpha = 0.6$) as for the commensurate island $N = 19$. However, the evolution with T of the diffusion amplitudes differ according to N and a detailed analysis of these differences proves that it is greatly influenced by the relative inter-shell movement. Indeed, we showed in a previous paper that for $11 \times 10^{11}\,\mathrm{K} \leqslant T \leqslant 12 \times 10^{11}\,\mathrm{K}$, the relative shells' movements are small in the $N = 19$ system [28]. In the diffusion experiments, one can observe that, in the same range of temperature, the diffusion curves are remarkably roughly superposable (fig. 4). Likewise, at higher temperature, we observe a stronger diffusion as the inter-shell movement is increased. Furthermore, for the same temperatures, the shells for $N = 18$ are always unlocked [28] and the amplitudes of the inter-shell displacements grow with temperature. Similar behaviours can be observed on the diffusion curves: whatever the temperature, the corresponding diffusion curves are always well separated and the diffusion amplitude increases also with the effective temperature (fig. 4). Then we can conclude that there exists a strong correlation between the inter-shell movement and the diffusion in each shell.

However, it would be naive to picture the two shells as a set of toothed wheels, the correlation effect being undoubtedly more subtle. Indeed, such a representation could suggest that a more important diffusion increase would be obtained in the case of two commensurate shells but this assumption does not correspond to the observations. Indeed, whatever the effective temperature, diffusion for the balls of the $N = 18$ island is always higher than for $N = 19$, the maximum difference being precisely at the temperature at which the $N = 19$ system is locked whereas the $N = 18$ system is unlocked (fig. 4). When the temperature increases, the differences are less important, the file of particles can be considered as floating above the modulated potential. It then only feels an averaged effect where the influence of the peculiar symmetries is hidden. We stress that a similar effect of the commensurability can also be observed for balls in the inner shells. Thus, even though the fluctuations of the modulated potential induces impulsion transfers, a better transfer is obtained in incommensurate configurations, a too good matching of the periods inducing the locking of the whole system rather than a "positive" cooperation.

Conclusion. – The diffusion of a file of particles can be accelerated when it moves in a quasi-1D fluctuating modulated potential. This enhancement presents two strong characteristics. The diffusion amplitude in a fluctuating potential is much larger than in a bald pipe. This effect seems to be favoured by the incommensurability between the modulation of the potential and the periodicity of the moving file. Moreover, the orthoradial m.s.d. increases as a power law t^α with $\alpha > 0.5$ whereas the diffusion in a circular simple file is characterized by $\alpha \leqslant 0.5$. This effect could be very important in the case of mobile vortices moving between pinned ones in superconducting devices. In particular, it has to be taken into account to design well-adapted patterns of traps in order to optimize the critical current. From the theoretical point of view, getting a better understanding of this phenomenon would require to develop a complete SFD theory including a fluctuating lateral potential.

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