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Relaxation and ordering processes in "macroscopic Wigner crystals"

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Abstract. The equilibrium configurations of a macroscopic Wigner crystal (2D system of interacting charged balls, mechanically excited) and their evolution towards these equilibrium configurations are presented. In particular, the variations of the number of remaining dislocations at equilibrium according to the number of particles, confinement shape and temperature have been extensively explored. One important result is the exhibition of the rapid creation of an unique grain boundary and its shrinkage during the annealing.

PACS. 68.65.-k Low-dimensional, mesoscopic, and nanoscale systems: structure and nonelectronic properties – 73.21.-b Electron states and collective excitations in multilayers, quantum wells, mesoscopic, and nanoscale systems

In the past years, many significant studies have been dedicated to 2D systems of interacting particles. Among them, we can mention systems like vortices in superfluid He^4 [1,2], electron dimples on a liquid helium surface [3], vortices lattice in mesoscopic superconductors [4], electronic configurations in semiconductor quantum dots [5], vortices in a Bose-Einstein condensate [6], trapped ions cooled by laser techniques [7], dusty plasma [8]. All of these experiments can be modelized by a system of particles interacting with an electrostatic potential. On the other hand, many numerical works have been also published to describe the ground and metastable configurations of such systems. Since an early theoretical study performed by Thomson in order to describe the atomic structure [9], various studies have been done involving coulombic [10–13], screened coulombic [13,14], or logarithmic interacting potential [9,13,15]. The influence of the confinement potential has also been explored [11,13].

In this work, extending our previous results to mesoscopic systems [16,17], we present the behavior during its way towards equilibrium configuration of a large elastic crystal consisting in electrostatically-interacting charged balls of millimetric size freely moving on an horizontal surface, the number of balls being larger than 1000 and the temperature being simulated by a mechanical shaking of the confinement cell. Underline that these studies are performed at room temperature with macroscopic particles which allows experiments significantly easier than those performed on others elastic systems.

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After a rapid description of the experimental set up in Section 1, we present rapidly the observed equilibrium configurations of this 2D elastic system in Section 2. However, the aim of this article is the presentation in Section 3 of the dynamical behavior of topological defects which is an important result, both from a fundamental point of view as well for more applied aspects of material science. Throughout the annealing process, the initial topological defects organize themselves by local motions and recombining in order to reduce the initial disorder. We will focus our discussion on this evolution which leads the system from its initial disordered state to a regular macroscopic Wigner crystal reached at equilibrium.

1 Experimental methods

The experimental set up has been previously described [16]. Basically, the system is constituted by a monolayer of about a thousand metallic balls (Fig. 1). The balls are placed between the electrodes of an horizontal capacitor and move directly over the bottom capacitor electrode. The potential between the electrodes is noted V_e. Moreover, the balls are laterally confined inside an isolated metallic frame introduced between the capacitor electrodes; the potential V_c applied on this frame can be varied in order to modify the electrostatic confinement. In the experiments described here we have used $V_e = V_c = V$. Experiments were conducted in various shaped frames. Whatever these shapes, it always exists for a large system ($N > 1000$) a central part of the crystal at equilibrium which is an hexagonal array. The higher the cell

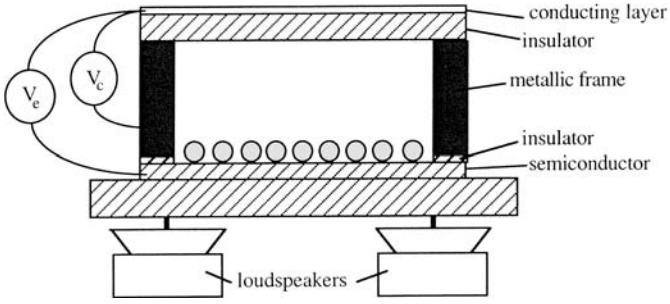

Fig. 1. Experimental set up.

dimension, the larger the central monocrystal size. Thus, taking advantage of this effect, we chose generally an hexagonal-shaped frame to optimize the size of the patterns with triangular symmetry. In the experiments presented there, the size of the frame edge is 400 mm, allowing inter-particle distance about 2 mm.

When the potential V of a few thousands volts is applied between the capacitor electrodes and maintained over the experiment duration, the balls get instantaneously a mono-disperse charge and repel freely each other to spread in the whole available space. The annealing process is then engaged. In order to obtain reproducible arrays and relevant data, the following procedure is systematically used: the cell is continuously shaken using two independent loudspeakers supplied by two white noises of range 0–200 Hz. This shaking produces random motion of balls which simulates a thermal Brownian motion. In order to check this point, we have studied the transition between the stable and the excited states of small systems ($N < 10$) and verify that the Boltzmann law is satisfied (a complete presentation of these results is in preparation). We can then conclude that shaking can be associated to an annealing performed at a constant effective temperature.

In order to record images of the balls arrays in real-time, a CCD camera is positioned over the top capacitor electrode which is a transparent conducting glass. This camera is connected to a computer, allowing us to locate the balls centers by finding peaks of intensity for each image. Once the center locations are known, different relevant quantities are extracted. In order to identify the short-range order, Voronoi constructions (patterns formed by the perpendicular bisectors of all segments joining neighboring balls) are performed in order to characterize the obtained arrays and their defects. In this representation a regular site corresponds to an hexagonal cell whereas the defects correspond to non hexagonal polygons. Let us finally indicate that the long-range order could also be characterized by the translational and orientational correlation functions, however this article is stressed on the local order and we shall not discuss these aspects here.

2 Description of equilibrium configurations

Just after the application of the potential V between the electrodes, when the annealing begins, the system is strongly disordered. This initial state is far from the equilibrium. Shaked at slow effective temperature (defined by a ball displacement $\Delta r/a < 0.05$ where a is the interball distance), the system explores its configuration space in order to find its minimum of energy. According to the experimental conditions the ground state is reached after a time varying from a few minutes to several hours. Whatever the experimental set up, this final state corresponds to a roughly regular lattice with some remaining topological defects. In systems with short range inter-particle interaction like colloids systems, the description of the geometrical structure in terms of nearest neighbor bonds has demonstrated its physical sense; we suppose here that this procedure is also relevant in our case in which previous studies have suggested a long range inter-particle interaction [16,17].

In this frame, the creation of a point defect involves the breaking of bonds and the creation of miscoordinated particles. In a perfect hexagonal array, the simplest defect is a disclination which corresponds to 5 or 7 fold sites which can be seen as a removed or added $\pi/3$ wedge in a perfect hexagonal crystal. However, isolated disclinations with opposite topological charge can attract each other via their strain field to form dislocations [18]. The edge dislocation (5–7 fold disclinations pair) which corresponds to a system of two additional half extra rows terminating on the 7-fold site is the most frequent defect in the case of coulombic inter-particle interaction [19]. Moreover isolated dislocations can interact and move in order to form complex defects. Some of these arrangements preserve both translational and orientational order and are associated to a slight distortion of the hexagonal lattice free of defects. Thus pairs of dislocations with opposite Burger's vectors are virtual defects. Associated to slight distortion, their energy cost is very low and they can easily be thermally excited and are frequently observed [18]. By contrast, more complex arrangements as lines of defects called "grain boundaries" are also observed to form polycrystalline systems (we return to this point in Sect. 3).

The number of remaining defects in the system depends on the symmetry of the confinement frame, the number of confined particles and the effective temperature since they are either resulting from the stress imposed by the confinement or thermally-induced. In a circular frame, these defects originate from the incompatibility between the symmetries of the ordered triangular lattice and those of the confinement which forbids a perfect hexagonal crystal and leads always to lattice defects whatever the number of balls. From the geometrical point of view, the irreducible number of disclinations in this case is determined by the Euler's theorem and six disclinations are required to adapt the triangular and the circular symmetries in 2D system. Experimentally the defect number is actually larger than 6 but can be reduced by annealing procedure. This result has been confirmed numerically for a system of electrons, the authors [20] indicating that these defects are inherent to the circular confinement and that their number cannot be reduced beyond a minimum value. By contrast, the symmetries of an hexagonal frame are compatible with

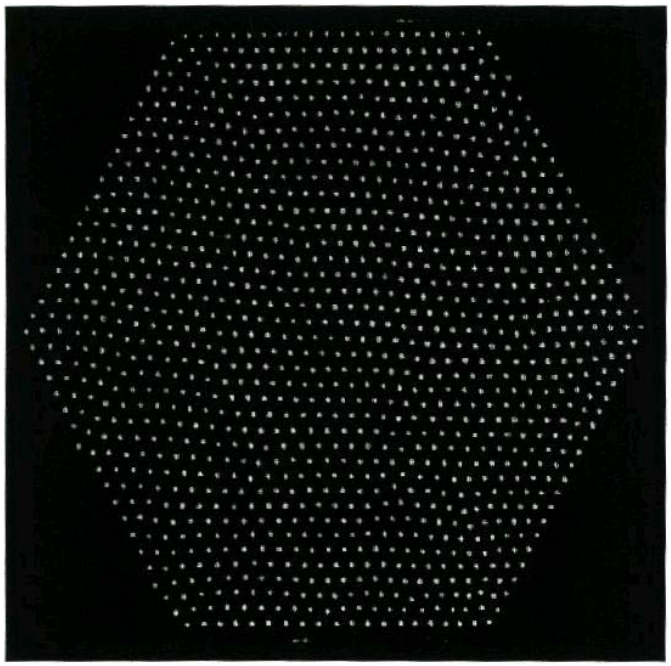

Fig. 2. Equilibrium configuration of a crystal constituted by a magic number of balls, $N = 1027$.

those of a triangular lattice and offer the best opportunities in order to optimize the exploitable size of the observed crystal. Thus in the following we retain this shape and we discuss results obtained with it.

In an hexagonal frame, perfect confined crystal can be expected. However, in this case, the number of residual dislocations strongly depends on the number of the confined balls. For "magic numbers", $N = 1 + 3p(p-1)$ [12], the confined packing is in agreement with an hexagonal symmetry and the crystal can be then defect-free at low temperature. Figure 2 is an example of such a crystal obtained for $N = 1027$. In order to evaluate the translational and rotational orders of this crystal, we have calculated the translational and rotational pair correlation functions associated to the particles positions and compared them to those evaluated for an finite ideal triangular lattice with the same size. The comparison presented in Figures 3a and 3b proves the complete agreement between the ideal array and the experimental one. These correlation functions exhibit well defined characteristic features; in particular the high first peak which corresponds to the nearest-neighbors indicates that the lattice parameter is well defined. By contrast, for non-magic numbers, the incompatibility of boundaries conditions with the formation of a regular pattern results in a small number of dislocations which remain whatever the annealing duration. This behavior is well described by Figure 4 which presents the mean number of 5-fold and the 7-fold sites observed at equilibrium in systems constituted of various balls number. For each number, the data are carried out from five equivalent systems annealed during 30 mn. Underline that, in order to obtain a coherent set of comparable

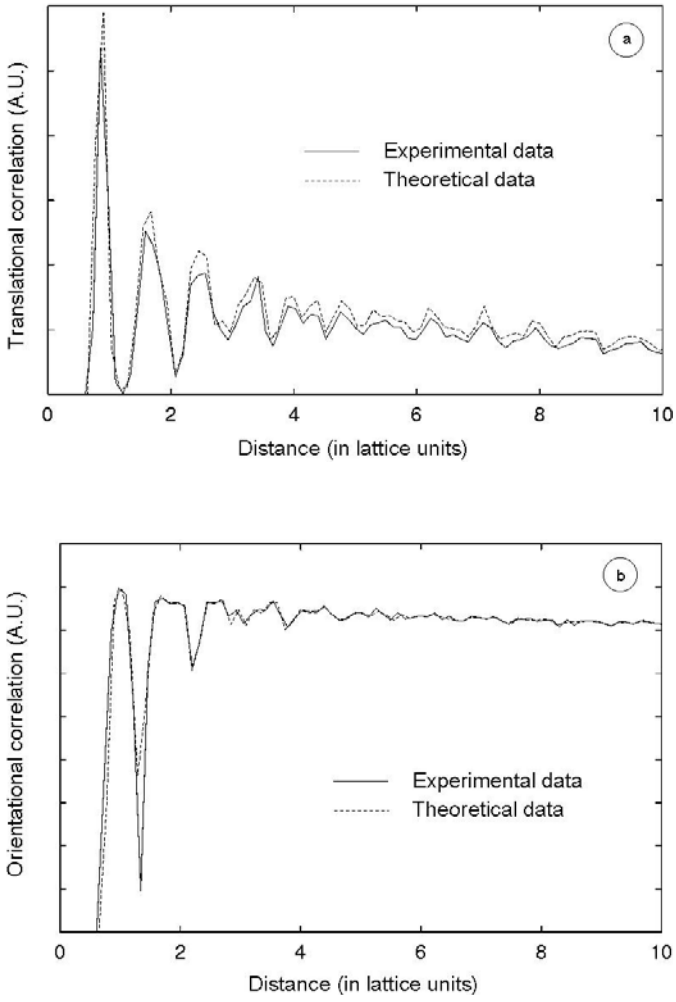

Fig. 3. Correlations functions obtained for a $N = 1027$ balls crystal; (a) translational correlation function, (b) rotational angular correlation function. The solid lines correspond to the correlation function obtained for an perfect crystal with the same size.

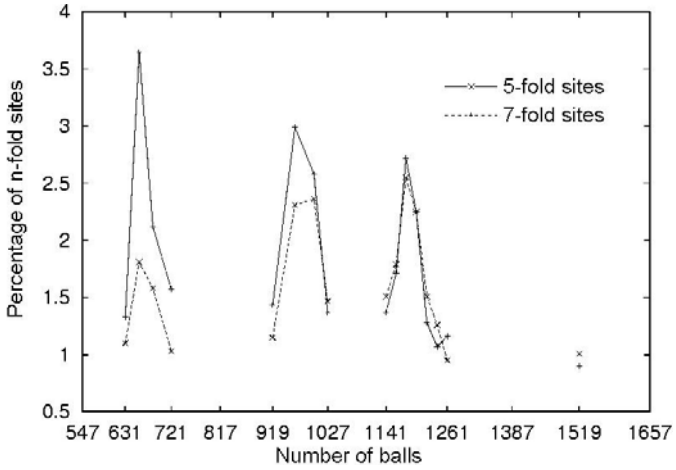

Fig. 4. Variations of the dislocations number with the number of balls. The magic numbers are indicated on the X-axis.

data, the amplitudes of position fluctuations (noted Δr) have to be kept constant whatever the number of balls; so we have modified the effective temperature T according to the number of balls in order to maintain the ratio of E/kT constant where kT is the effective thermal energy and E the electrostatic energy which varies with the number of confined balls via the inter-ball distance. This curve exhibits minima for the magic number system and maxima of defects for systems constituted with a number of balls chosen far away from two consecutive magic numbers. These variations can qualitatively be interpreted. Let us start from a magic number for which the defect number is minimum and increase progressively the number of confined balls by adding new balls. Initially, this addition is associated to an increase of the number of dislocations, located near the frame edge. These added balls result in apparition of new extra rows in the array. At the beginning, these rows are short since the added ball number is small. As this number increases, we observed that the system prefers to elongate the extra rows rather than create a set of new dislocations. This is confirmed by the evolution of the mean length L of the extra rows. For instance, between the two consecutive magic numbers 1027 and 1141, the add row length L corresponding to $N = 1050$, 1063, 1087 is respectively 8.9, 10.4, 14.6 lattice spacing units. This preference explains the observed decrease of the percentage of dislocations as the ball number N increases. We can also remark that the number of 5-fold sites is generally higher than those of the 7-fold and the relative quantities of 5-fold increases slowly whereas the 7-fold ones increases. Even if we cannot explain these results, it is in agreement with the numerical studies performed for smaller systems constituted by a few hundred of balls [21].

Lastly, in order to evaluate the respective quantities of defects due to the confinement stress with respect to those thermally-induced, we studied the equilibrium ground configurations reached for different values of the effective temperature. At high temperature ($0.1a < \Delta r < 0.2a$), the array contains a large number of free mobile dislocations or dislocations arrangements and isolated disclinations have seldom been observed. At lower temperature, dislocations are still observed however their number and their mobility are smaller. Let us indicate that this decrease is regular and efficient up to a very low temperature ($\Delta r = 0.05a$), at which the number of residual dislocations then remains constant. At these low temperatures, this irreducible number only depends on the number of confined balls and on the symmetry of the frame.

In order to complete these qualitative results, it is convenient to precise the final location of the remaining dislocations at very low temperature. Experimentally, this question is very sensitive to the annealing process. Some generic behaviors, nevertheless, can be exhibited. According to the nature of the inter-ball interaction and to the shape of the confinement potential, various different situations have been observed. When the balls move on the bottom electrode of a capacitor and are confined in an hexagonal frame, the remaining dislocations are very mobile. These conclusions result from the observations of "mean crystals" obtained by superposing the positions of balls recorded during the annealing: in this case, the balls in sites appear as stable balls whereas the balls near the dislocations are much mobile and appear as diffuse hallow in the mean crystal. The positions and the mobility of the residual dislocations are very different for a circular frame. In this case, the residual dislocations are roughly static and located very close to the confinement surface, in the few outer shells of balls. Lastly, let us indicate that when the charged spheres are free to move over an insulating surface and are confined in a circular frame, the center and the edge of the crystal are free of defects when equilibrium is reached, the residual defects being mostly located inside the system where they seem to be trapped. This kind of behavior is observed whatever the number of balls.

Before to describe the relaxation of these topological defects, we can give some comments about equilibrium state. From the theoretical point of view, the observed crystalline state obtained when the equilibrium is reached is in agreement with the triangular array with quasi long range order predicted as the most stable structure for an infinite 2D system of long range interacting particles [19,22]. In such isotropic elastic 2D systems, a discussed scenario of two phase transitions assisted by defects has been proposed by Kosterlitz and Thouless [23]. In this model, a perfect quasi-long-range translational order, without defect or with virtual dislocations [24], can be observed at low temperature. When the temperature increases, this translational order is broken when the temperature reaches T_c but the orientational correlations are not destroyed and persist until a second temperature T_i, at which the system gets in its liquid phase. The phase between T_c and T_i is named the "hexatic phase". In our system, this scenario of two successive phase transitions has never been observed. In particular the melting of the crystal corresponds to a strong and rapid increase of the dislocations number with the temperature. The dissociation of the dislocations in isolated disclinations has not been observed. Arguments of different nature found in the literature can be used to explain this result. The first one suggests that the finite dimensions of a system would result in a shift of the solid-hexatic temperature in order to join the hexatic-liquid transition [25]. On another hand, the literature suggests that in the case of 2D system with long range interacting particles, the temperature T_c could be very high. Indeed, if we retain the common assumption that the expression of the critical temperature T_c obtained in the K.T. theory could also describe the melting transition for 2D system of long range interacting particles, we obtain an infinite T_c since this temperature is proportional to the Lamé coefficient λ of the system which is infinite for a 2D electron solid [19]. Underline nevertheless that this common assumption is not obvious since the K.T. scenario has been developed only for short range interaction. Similar effect could be expected in our system for which the inter-ball interaction seems to be long range interaction as it can be inferred from our previous experimental results [16].

In this framework, the absence of transition could result from an increase of the critical transition temperatures, the T_c transition being the only one which could be observable as suggested by the small number of isolated disclinations observed at the experimental effective temperature. An alternative argument has been recently proposed in a theoretical study [26] which suggests that in the case of a system of particles interacting via a logarithmic potential the system would remain liquid at any temperature. However the correlation length varying as $(1/T)^{1/2}$, a system with finite size would appear as a regular array, with a triangular ground state and a very small number of disclinations at low temperature. These comments show that the origin of our observation is not yet clear, let us indicate however that the existence of K.T. transition for systems with short range interaction like colloid systems is also experimentally contested [27].

Our results obtained about the dislocations locations are in agreement with those predicted in numerical studies. For a circular confinement potential, Ying-Ju Lai et al. [21] have numerically shown that for the case of logarithmic interaction the array has a quasi uniform packing density distribution and the residual defects are essentially located in the outer part of the system, near the confinement frame. On the other hand, in the case of coulombic potential, all the dislocations are observed in a ring of width over a few inter-ball distance from the confinement edge and roughly form a stable grain boundary of connected dislocations; the inner and outer balls being respectively located on a quasi triangular lattice and on a bend triangular lattice. These results were numerically confirmed by Bedanov et al. [11]. More precisely, the defects do not come close to the confinement frame. Indirectly, this qualitative agreement confirms our previous conclusions about the nature of the inter-balls interaction. It seems then that the inter-balls interaction is coulombic or logarithmic whether the balls are placed on an an insulator or an conductor [16].

Lastly, the large defect mobility observed in the case of hexagonal confinement could be associated with this particular geometry. Indeed, in this case the repulsive elastic interaction between a dislocation and its image in the frame is logarithmic since the hexagonal confinement stands for an infinite crystal; in the same time, the real electric charge associated to this dislocation interacts with its electrostatic image also by a logarithmic interaction. The distance dependence of these two interactions being identical, the dislocations cannot find a location corresponding to a minimum of energy and then can move in the whole array. This particular situation is characteristic of the hexagonal confinement and falls down as soon as the confinement presents other symmetry. This point has to be clarified.

3 Grain boundary dynamics towards the equilibrium

In the previous section, we described the ground configuration observed when the equilibrium was reached. Let us now describe the array evolution during the annealing process. In particular, we stress the dislocations behavior and their reorganization leading the system from an uniform topological defects density to its final regular ground state. We restrict our discussion to results obtained with a system containing a magic number of balls free to move on a conductor and confined by an hexagonal frame in order to obtain a final state free of defects.

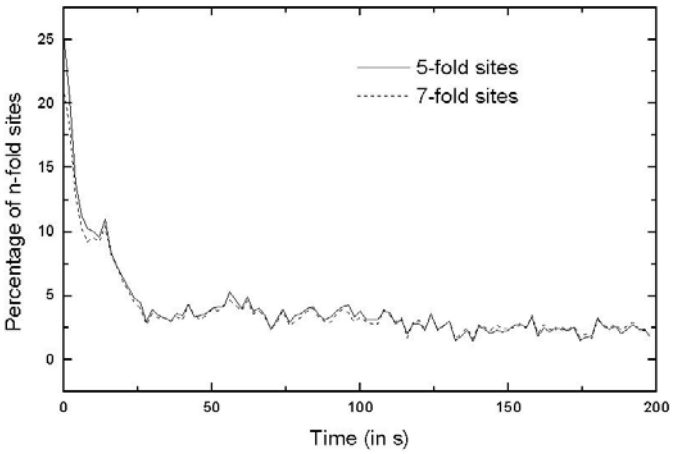

Fig. 5. Evolution with annealing time of the number of dislocations in the case of magic number system. The error bar is less than 1%.

In plastically deformed crystals, the accumulation, motion and interaction of a large numbers of dislocations give rise to complex dynamics which results generally in dislocations spatial patterns. In our case, this evolution consists schematically in two successive stages which can easily be identified by observing the evolution of the number of 5-fold and 7-fold sites with the annealing time. In Figure 5 we present this evolution obtained from different runs performed on a system with $N = 1027$ but similar lots can be observed for any magic number. During the first tens of seconds, we observe a very fast decrease until over 50% of the number of 5-fold sites followed by a slower reduction for longer shaking time. The quasi-totality of the dislocations vanishes after an annealing time about 500 s. These two different characteristic times suggest two different mechanisms resulting in the reduction of the defects number. More precisely, when the electrification potential V is applied between the electrodes the balls being unshaken, the initial array is characterized by a large quantity of defects randomly dispersed. At this initial stage, the system is very disordered, largely far from its final regular configuration and its stored energy is associated to elastic deformations due to the defects. The majority of these observed defects are dislocations; we are thus allowed to suppose a very expensive energetic cost for isolated disclinations. This high cost could be due to the confinement and the range of the interaction since isolated disclinations can be observed in short range interacting colloid systems. Thus, in order to minimize this deformation energy, disclinations with opposite topological charge strongly attract

each other and quasi-instantaneously form dislocations which give the main contribution to the stored energy.

However, number of these dislocations are topologically useless and the small stored energy seems sufficient to induce the dislocations movement observed at the first times of the annealing ($t < 20$ s). Indeed, the internal stress experienced by a given dislocation depends on the configuration of the other dislocations in its surrounding; so, in order to reduce the deformation energy, the dislocations are very rapidly eliminated after a short annealing time either by evacuation through the crystal edge, or annihilation process of dislocations with opposite Burger's vectors or reorganization after glide or climb processes. Generally, dislocations easily move in the glide direction perpendicular to the direction joining the 5–7 fold disclinations but less easily in the climb direction parallel to this direction because climb involves absorption and emission of vacancies and interstitials while glide does not. However in our case the dislocations movement is more free since the energy associated to dislocation climb is small as suggested by the increase of add row lengths. Moreover, our system of balls is confined and is then equivalent to a bi-crystal constituted by an elastic array embedded in an rigid lattice characterized by infinite elastic constants; so, due to the repulsive image force of dislocation on the frame edge, the glide or climb are equivalent and their energies can be considered as roughly equal. Thus, as the movement of the dislocations is easy and isotropic, the recovery is efficient.

During the beginning of the annealing ($t < 20$ s), the isolated dislocations move and interact to rapidly form short dislocations lines which organize themselves in order to constitute continuous grain boundaries (Fig. 6). The system appears then as polycrystalline, each crystallite containing roughly 200 balls. Inside these grain boundaries, the arrays are triangular lattices but misorientated with respect to the confinement axis; by contrast, the outer grain array corresponds to a regular triangular lattice oriented along the axis of the confinement frame. Let us indicate that we never observed the direct recrystallization characterized by the growth of a grain (germ) in the disordered system. Such a "polyganized" phase, well-known in metallurgy, is generally very stable, the boundaries corresponding to small misorientation. However, in our case, surely due to the confinement, these grain boundaries remain largely misoriented and then are unstable. So they move in order to reorganize themselves to form a unique grain boundary which is reached after a few tens seconds of a slow annealing. During this annealing, we systematically observed a faster reduction of domains with the larger misorientation (with respect to the frame orientation). In particular, when two neighbor domains have the same surface but different misorientations, the less misoriented is favored, its surface increases whereas the surface of its neighbor reduces. This indicates that the energy associated to the boundary depends on the relative misorientation of the domains. Moreover, for two adjacent domains of different sizes but with the same misorientation, the boundary moves in a sense which extends the larger domain. This phenomenon looks like the "coarsening phase" well-known in metallurgy; however, on the contrary to the usual coarsening behavior, our system prefers to eliminate the grains with small curvature radius and to increase the grain surface. These two observations suggest that in our confined systems, the boundary energy depends not only on the dislocations line length and its orientation but also on the whole surface of the domain. This effect could be due to the long range character of the interball interaction. After this initial reorganization, the system appears as constituted by an unique grain, roughly circular, embedded within a large parent grain well oriented with respect to the confinement frame. Let us indicate that whereas this unique grain boundary is always observed in our systems, they are not observed in systems with short range interaction [28]. Once more, this suggests that this dislocation pattern depends on the range of the interaction.

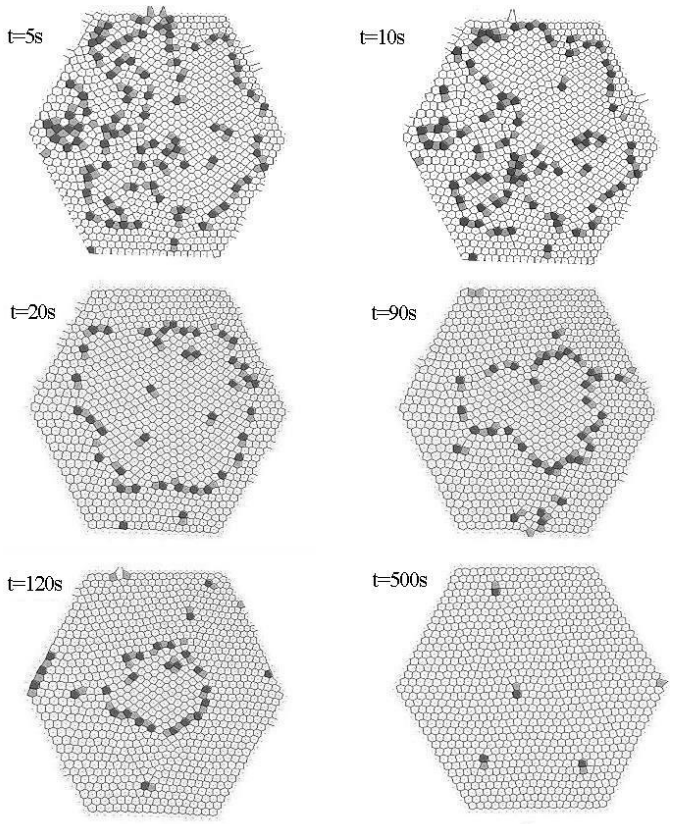

Fig. 6. Evolution with time of the grain boundary. The dislocations are visualized by attributing special colors to such defects (pentagons are filled in dark grey, and heptagons in bright grey).

From these observations we can conclude that the first stage observed in Figure 5 corresponds to the time, about 20 s, required to form this unique grain boundary. The short duration of this initial process indicates a high mobility of the dislocations associated to small energetic barriers. This time is correlated to the friction of the balls on the surface as it is confirmed by experiments performed on system shaken before the application of the potential V.

In this case, the activation energy is such that the intermediate dislocations short lines cannot be observed and that the configurations with dislocations grain boundary are apparently instantaneously obtained.

The system reaches then the second phase of its evolution. As the annealing is carried on, a shrinkage of the internal grain can be observed: the unique grain boundary moves in order to drive the boundary towards its center of curvature resulting in a perfect crystal without defects. Figure 6 presents this evolution. Since the characteristic dimensions of the grain boundary are always larger than the dislocation core and since the width is small, the boundary evolution can be analyzed in the framework of the elasticity theory which means considering the boundary as a line. For an infinite 2D system, a grain boundary is then characterized by two independent parameters: θ which describes the angular disorientation between the two domains, and the relative orientation α of the grain boundary with respect to a reference axis [18]. The energy per length unit of the grain boundary, noted γ, has been derivated in the case of short range interacting particles [29]. However, Fisher et al. have shown that the derivated expressions for γ are also valid in the case of 2D electron crystal. Thus we can then suppose that this approach is adapted to our case [19].

In these elastic theories, when the disorientation is low, a grain boundary is a sequence of dislocations regularly organized along a line. In this geometry, the stress field of a dislocation is annihilated by those of its two neighbors. Thus the energy stored in such a grain boundary is small and the linear defect configuration is very stable. For an infinite crystal, this energy is given by $\gamma(\theta) = \gamma_0\theta(A - \ln\theta)$ where γ_0 and A are constants. By contrast, when the two arrays are largely rotated one with respect to the second, there exist sites in coincidence, the line joining these coincidence sites lattice constituting a high angle boundary. With two hexagonal arrays the simplest energetically favored grain boundary corresponds to an angle of 38°. The corresponding stored energy cannot be evaluated analytically since some atoms near the coincidence grain boundary could be very closed and have to relax. The resulting energy is generally high and the boundary is unstable. In our confined systems the situation could be different. In particular, the distinction between these two kinds of extended defects is not obvious. Indeed, the interaction between the dislocations and the frame edge does not allow the development of a straight boundary grain. So it would be essential to introduce this confinement of the grain boundary in the evaluation of its energetic cost. A very simple model of this shrinkage can be built since a 2D hexagonal lattice may be consider as an isotropic elastic array. According to an assumption recently proposed by Kobayashi et al. [29] to describe the dynamics of the grain boundary in a circular bicrystal, let us suppose that the energy γ is constant and positive since the grain boundary moves through its curvature center inducing the decrease of the length of the grain boundary. The grain boundary being roughly circular, thus a simple one dimensional axisymmetric model can be used. If we assume an over-

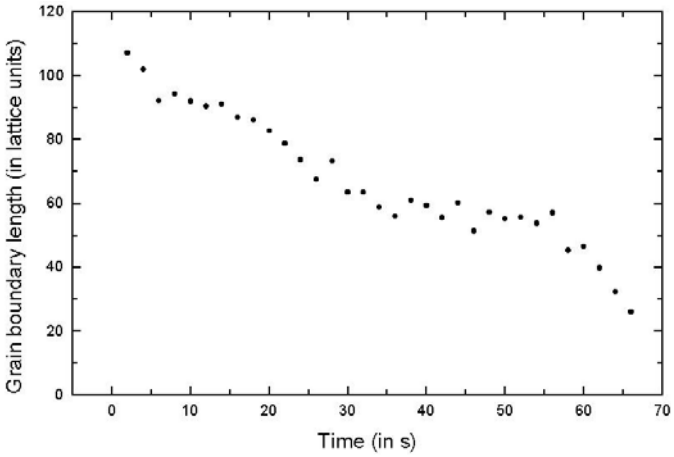

Fig. 7. Evolution of the grain boundary length with the annealing time.

damped motion of the grain boundary, the friction force is then linearly proportional to the grain velocity which is proportional to the product of the grain mobility M by the local force applied F. The mobility is thermally activated whereas the local force results from the curvature of the grain boundary and is equal to $F = 2\gamma/R$ where R is the radius of the grain

$$dR/dt \propto 2\gamma/R.$$

If we consider γ as independent of R, the solution of this equation gives $R(t)$ equal to $(R_0^2 - 4k\gamma t)^{1/2}$ where R_0 is the initial radius and k a constant. Our observations do not seem in agreement with this behavior. In order to compare our data, we have evaluated $R(t)$ by measuring the variation of the length of the grain boundary with time and considered that $R(t)$ is proportional to this length as for a circular boundary. A complementary estimation have been done by measuring the surface inside the grain boundary and taking the square root of this surface. The two estimations are similar. In Figure 7, we present the variations of $R(t)$ obtained by the first estimation. On this curve, $R(t)$ looks more like a linear function of t. This linear decrease of the grain boundary length with the annealing time could be explained by an energy γ proportional to the curvature radius or to the boundary length, which is equivalent in the case of a circular boundary. The origin of this dependence could be associated to the long range of the inter-particles interaction. This assumption could be validated by the fact that the shrinkage of an unique grain boundary has not been observed in systems with short range interaction as colloids or magnetic bubbles.

4 Conclusion

The equilibrium configurations of a macroscopic Wigner crystal constituted of a 2D system of interacting charged balls mechanically excited by loudspeakers have been studied. In particular, we have extensively explored the variations of the number of dislocations remaining when

the equilibrium is reached according to the number of particles, confinement shape and temperature. We have shown that for systems constituted with a magic number of balls, the array at equilibrium is a triangular lattice without defects, excepted those thermally excited.

The evolution of the array of a macroscopic Wigner crystal towards its equilibrium configuration has also be studied. One important result is the exhibition of the rapid creation of an unique grain boundary and its shrinkage during the annealing. The long range interaction between particles could contribute to this particular behavior, never observed in others 2D systems.

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